Laser in stationnary state
Continuous Wave (CW) laser

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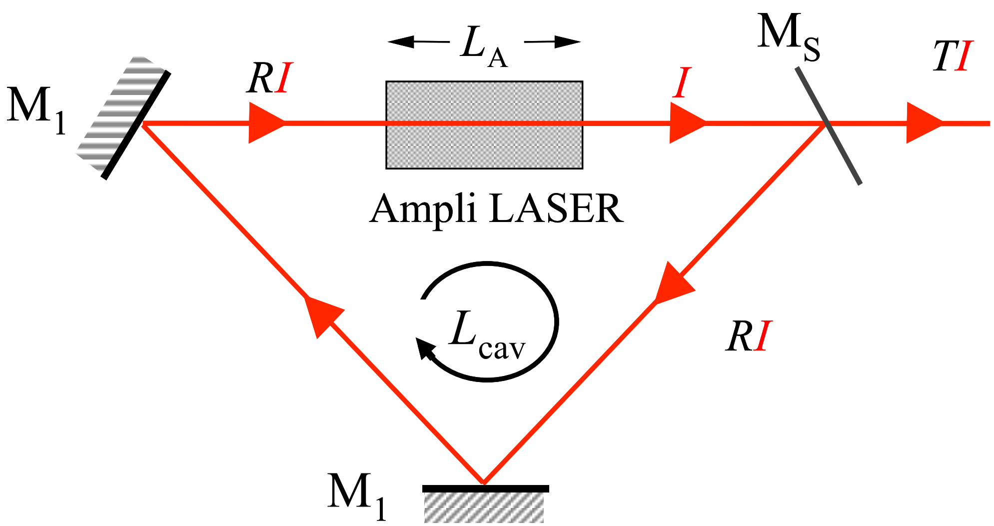
   - Homogeneous and inhomogeneous broadening
   - Multimode emission; simple or bistable mode competition

E. Conclusion
   - Generality of the behaviors

Laser oscillation



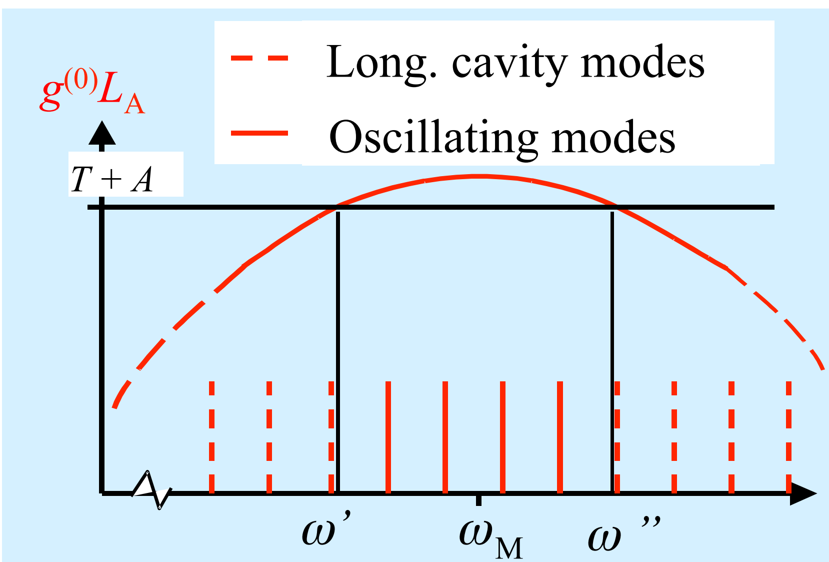
Gain condition

$$\omega' < \omega < \omega''$$

Phase condition

$$\omega_p = p\, 2\pi \frac{c}{L_{cav}}$$

p integer number



In lecture 4 :

Quantitative treatment of the laser behaviour:

steady state (stationnary) solution

A priori a very difficult problem:

- Field: obeys partial differential equations
- Laser medium: non linear

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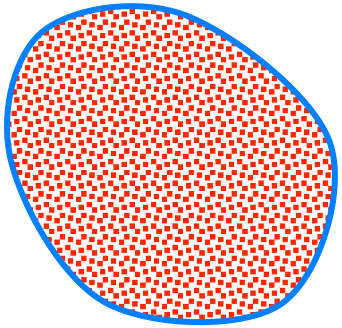
Modes of a closed lossless cavity



Complex field (Analytic signal)

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}^{(+)}(\mathbf{r},t)+\mathbf{E}^{(-)}(\mathbf{r},t) \quad \text{with} \quad \mathbf{E}^{(-)}=\left[\mathbf{E}^{(+)}(\mathbf{r},t)\right]^{*}$$

Totally reflecting boundary (perfect conductor) : expansion

$$\boxed{\mathbf{E}^{(+)}(\mathbf{r},t)=\sum_{p}\mathcal{A}_{p}\,\mathbf{u}_{p}(\mathbf{r})\,\mathrm{e}^{-i\omega_{p}t}}$$

$p=\{$3 integer numbers (3D) + 1 two-valued number (polarization)$\}$

$\boxed{\text{modes}}$ $\boxed{\left\{\mathbf{u}_{p}(\mathbf{r})\right\}= \text{ orthogonal normalized basis} \qquad \int d^{3}r\,\mathbf{u}_{p}^{*}(\mathbf{r})\,\mathbf{u}_{q}(\mathbf{r})=V_{\mathrm{cav}}\,\delta_{pq}}$

- Much simplified description of the field: discrete series $\mathcal{A}_p$ of complex numbers instead of a vector field
- Very simple simple time evolution
- Analogous to Schrödinger time depending solution: expansion on eigenstates of the Hamiltonian

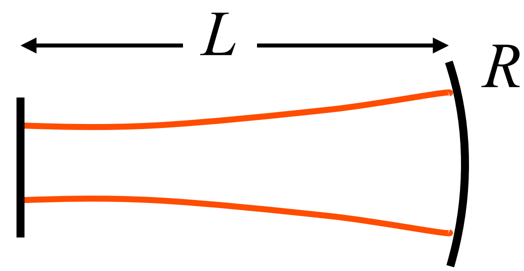
No approximation

Modes of a stable open lossless cavity

Stable lossless cavity

Ensemble of perfect mirrors leading to stationary solutions of the propagation equation (modes)

Condition on mirrors (position, orientation, curvature)



Example of a plane-concave cavity

Stable if $R > L$, mirrors aligned

Expansion on the modes of the cavity

$$\mathbf{E}^{(+)}(\mathbf{r},t) = \sum_p \mathcal{A}_p \, \mathbf{u}_p(\mathbf{r}) \exp(-i\omega_p t)$$

Marginal case $R >> L \Rightarrow$ quasi-cylindrical beam (quasi plane wave, diffraction is negligible): Fabry-Perot interferometer

$\Rightarrow$ Homogeneous (top hat) model

Homogeneous (top hat) mode model

Aligned plane mirrors

⇒ Marginally stable cavity

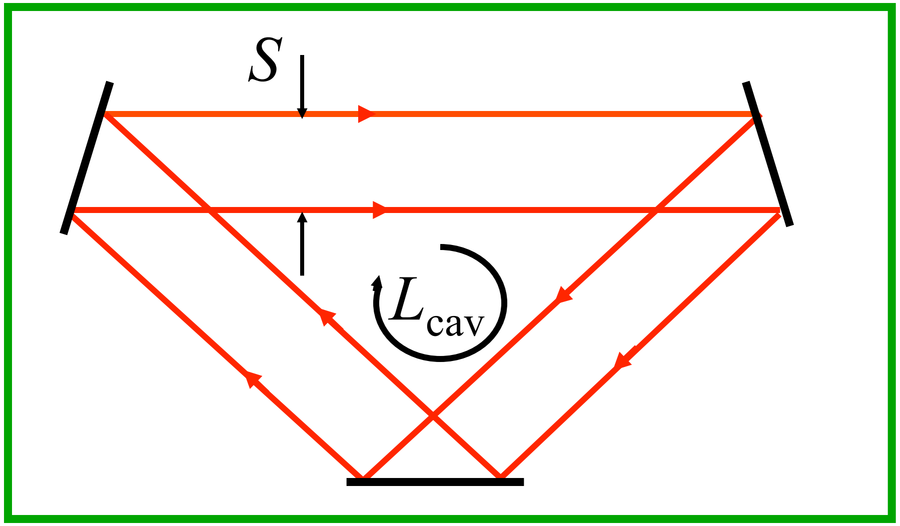
⇒ Mode = recycled plane wave

$$\mathbf{E}_p^{(+)}(\mathbf{r},t) = \sum_\alpha \mathcal{A}_p^\alpha\, \mathbf{e}_p^\alpha \exp(i\mathbf{k}_p^\alpha.\mathbf{r} - i\omega_p t)$$

⇒ no transversal structure

(only one integer $p = L_{\text{cav}} / \lambda$)

⇒ Constant amplitude: simplified calculations



Not a fully realistic model (diffraction as well as mirror curvatures determine the transverse structure) but a convenient model:

- Avoids tedious integrals over transverse profiles
- Allows us to grasp the basic physics, and to obtain correct equations, within numerical prefactors
- Easy to adapt to more realistic cases (Hermite-Gauss, Laguerre-Gauss, … normal expansions)

Homogeneous (top hat) mode model

Aligned plane mirrors

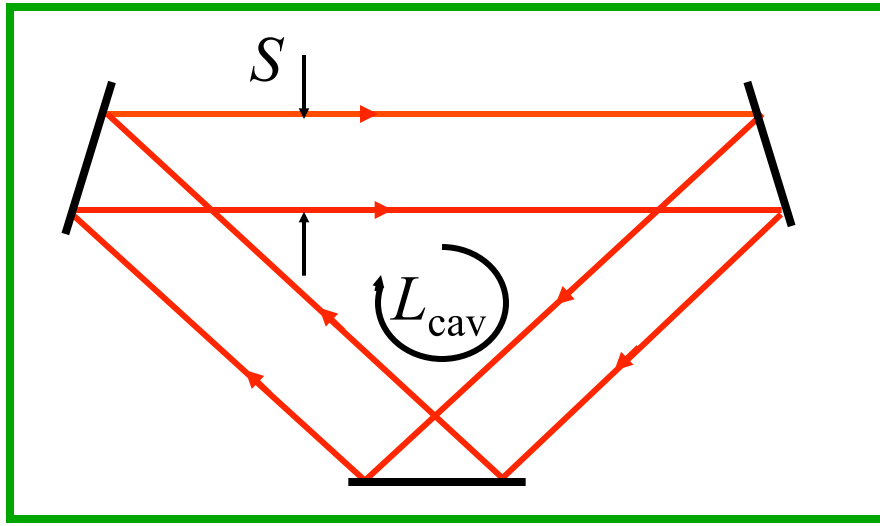
$\Rightarrow$ Marginally stable cavity

$\Rightarrow$ Mode = recycled plane wave

$$\mathbf{E}_p^{(+)}(\mathbf{r},t) = \sum_\alpha \mathcal{A}_p^\alpha \, \mathbf{e}_p^\alpha \exp(i\mathbf{k}_p^\alpha . \mathbf{r} - i\omega_p t)$$

$\Rightarrow$ no transversal structure

(only one integer $p = L_{cav} / \lambda$)



$\Rightarrow$ Constant amplitude: simplified calculations

Top hat mode model: simplified calculations

Finite transverse size: $S \Rightarrow V_{cav} = S\,L_{cav}$

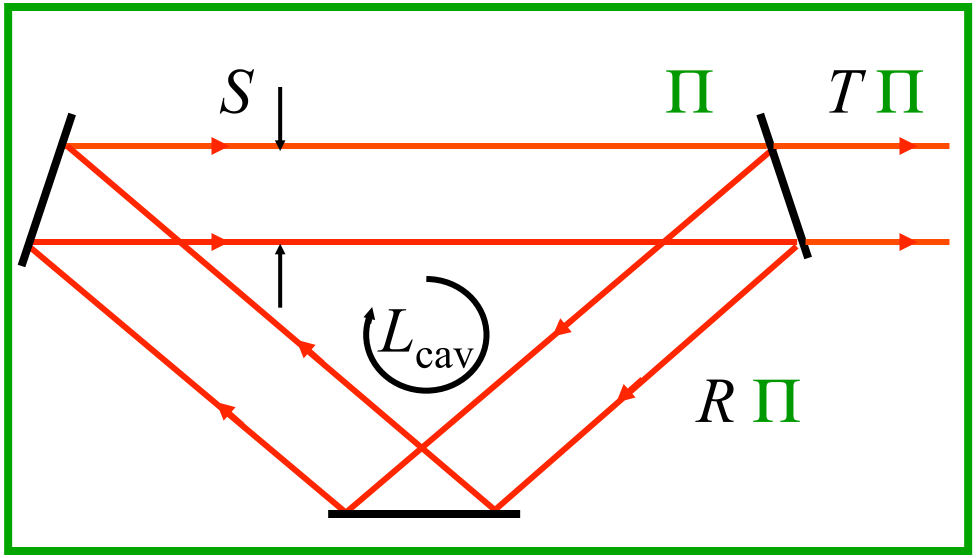
Number of photons in cavity

Energy in the mode $\quad U_p = \varepsilon_0 \overline{\mathbf{E}(\mathbf{r},t)^2}\, V_{cav} = 2\varepsilon_0 \left|\mathcal{A}_p\right|^2 V_{cav} = \mathcal{N}_p \hbar\omega_p$

Poynting vector $\quad \Pi_p = \varepsilon_0 c\, \overline{\mathbf{E}(\mathbf{r},t)^2} = 2\varepsilon_0 c \left|\mathcal{A}_p\right|^2 = \dfrac{U_p}{V_{cav}} c = \dfrac{1}{S} \dfrac{U_p}{L_{cav}/c}$

Round trip travel time

Lossy cavity: generalized modes



Weak losses ($T << 1$):

small over 1 turn ($= L_{\text{cav}}$ / c)

$$\frac{dU_p}{dt} = -T\Pi_p S = -T\frac{cU_p}{S\,L_{\text{cav}}}S$$

$$\frac{dU_p}{dt} = -\gamma_{\text{cav}}U_p \quad \text{with} \quad \gamma_{\text{cav}} = T\frac{c}{L_{\text{cav}}}$$

Expansion over generalized modes

$$\left[\frac{d\mathcal{A}_p}{dt}\right]_{\text{losses}} = -\frac{\gamma_{\text{cav}}}{2}\mathcal{A}_p \quad \text{with} \quad \gamma_{\text{cav}} \ll \frac{c}{L_{\text{cav}}}$$

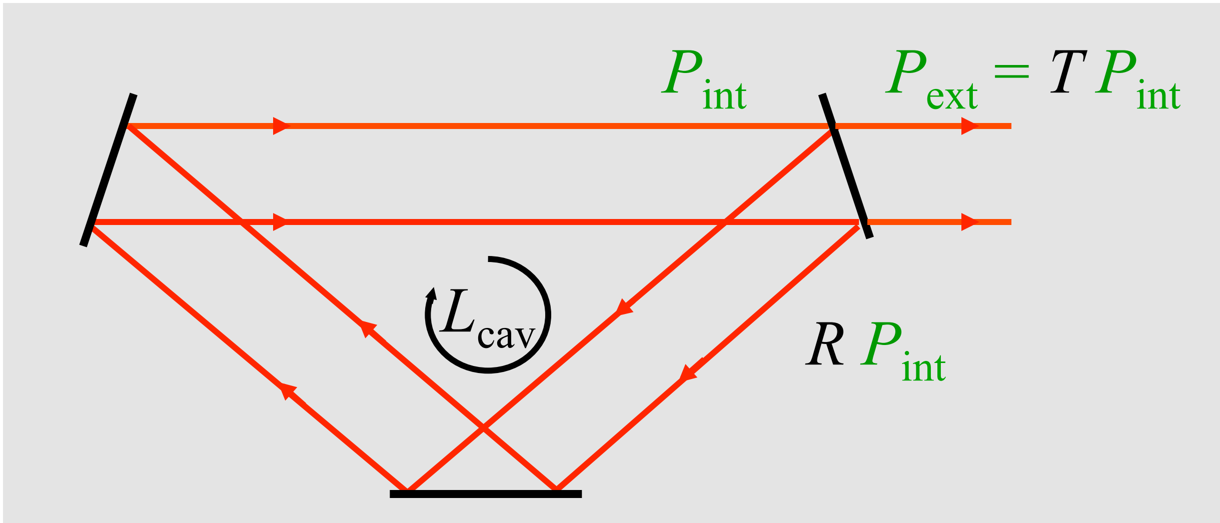
$$\mathbf{E}^{(+)}(\mathbf{r},t) = \sum_n \mathcal{A}_p(t)\,\mathbf{u}_p(\mathbf{r})\,\mathrm{e}^{-i\omega_p t}$$

$$\omega_p = p\,2\pi\frac{c}{L_{\text{cav}}}$$

Damping due to output coupler (T) and losses (α for 1 turn): lossless $T = \alpha = 0$

$$\gamma_{\text{cav}} = (T+\alpha)\frac{c}{L_{\text{cav}}}$$

Number or photons in a cavity mode vs. output power



$$U = P_{\mathrm{int}} \frac{L_{\mathrm{cav}}}{c} \qquad \boxed{\Rightarrow \quad \mathcal{N} = \frac{U}{\hbar\omega} = \frac{1}{T} \frac{P_{\mathrm{ext}}}{\hbar\omega} \frac{L_{\mathrm{cav}}}{c}} \quad \text{useful}$$

Example: He-Ne laser ($\lambda = 0.633\ \mu\mathrm{m}$)

$$P_{\mathrm{ext}} = 10\ \mathrm{mW}\ ;\ T = 10^{-2}\ ;\ L_{\mathrm{cav}} = 0.75\ \mathrm{m}$$

$$\boxed{\mathcal{N} = 2 \times 10^{10}}$$

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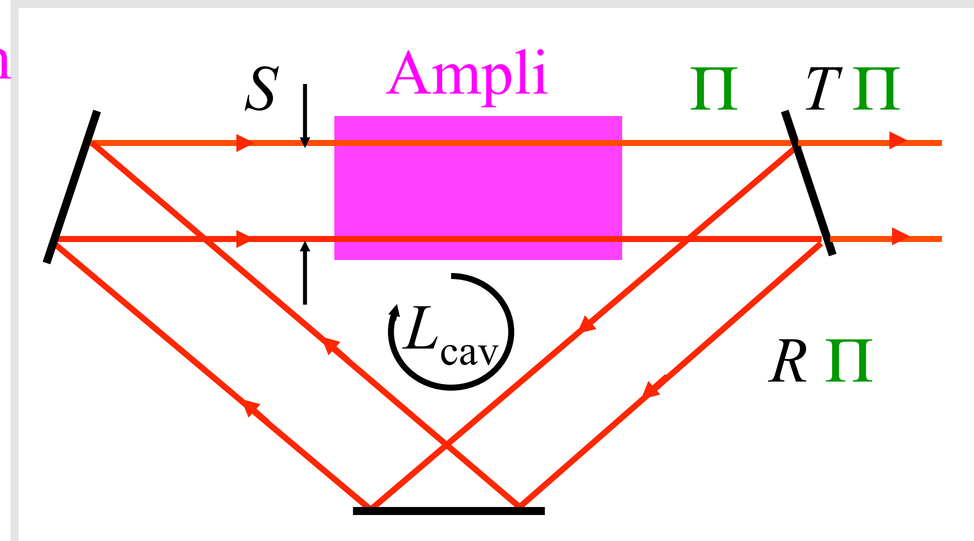
Cavity with laser amplifier: gain term



Propagation in amplifying medium

$$\mathbf{E}_{\text{out}}^{(+)} = \mathbf{E}_{\text{in}}^{(+)} \exp \frac{gL_{\text{A}}}{2} \exp\left\{ ik'L_{\text{A}} \right\}$$

Expansion on generalized modes:

$$\mathbf{E}^{(+)}(\mathbf{r},t) = \sum_p \mathcal{A}_p(t)\,\mathbf{u}_p(\mathbf{r})\,\mathrm{e}^{-i\omega_p t}$$

Evolution of a mode amplitude $\mathcal{A}_p(t)$

- Phase terms incorporated into $L_{\text{cav}} = L_{\text{géo}} + (n_{\text{A}} - 1)L_{\text{A}}$
- Gain par pass $\delta\mathcal{A}_p \simeq \dfrac{gL_{\text{A}}}{2}\mathcal{A}_p \quad \Rightarrow \quad \boxed{\left[\dfrac{d\mathcal{A}_p}{dt}\right]_{\text{gain}} = \dfrac{gL_{\text{A}}}{2}\dfrac{c}{L_{\text{cav}}}\mathcal{A}_p}$

Validity of this approach: atoms in a steady state (forced by the light field in the cavity) $\gamma_{\text{cav}} \ll \Gamma_{\text{atoms}}$

Gain saturation (reminder)

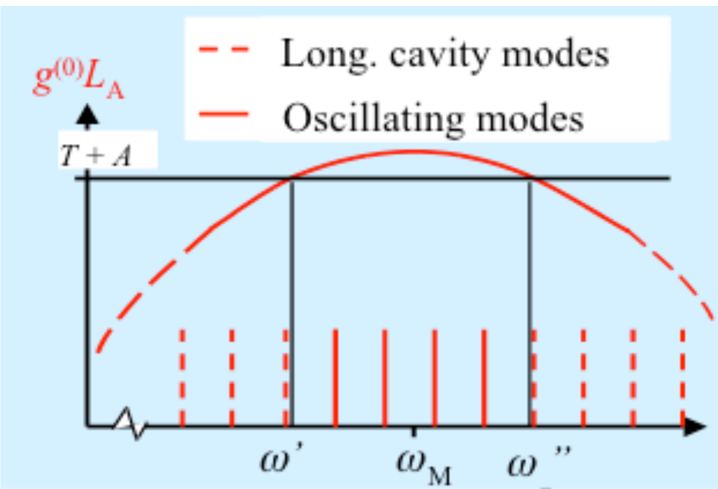
General expression (steady state)

$$\boxed{g(\omega) = \frac{g^{(0)}(\omega)}{1+s}} \quad \text{with} \quad g^{(0)}(\omega) = \sigma(\omega)\left[n_b - n_a\right]^{(0)}$$

Lorentzian variation

Non saturated gain

Non saturated population inversion



$$g^{(0)}(\omega) = g^{(0)}(\omega_M)\frac{1}{1+\dfrac{4(\omega-\omega_M)^2}{\Gamma_2^2}}$$

Mode p

$$g = \frac{g^{(0)}}{1+\dfrac{I}{I_{sat}}} = \frac{g^{(0)}}{1+\dfrac{2\left|\mathcal{A}\right|^2}{I_{sat}}}$$

$$\boxed{\left[\frac{d\mathcal{A}_p}{dt}\right]_{gain} = \frac{g^{(0)}L_A}{2}\frac{c}{L_{cav}}\frac{\mathcal{A}_p}{1+\dfrac{2\left|\mathcal{A}_p\right|^2}{I_{sat}}}}$$

Evolution equation of a laser mode
(amplifier in a steady state)

$$\frac{d\mathcal{A}}{dt}=\left[\frac{d\mathcal{A}}{dt}\right]_{\text{losses}}+\left[\frac{d\mathcal{A}}{dt}\right]_{\text{gain}}=\left\{-\underbrace{\frac{T+\alpha}{2}\frac{c}{L_{\text{cav}}}}_{\gamma_{\text{cav}}/2}+\underbrace{\frac{g^{(0)}L_{\text{A}}}{2}\frac{c}{L_{\text{cav}}}}_{r\,\gamma_{\text{cav}}/2}\frac{1}{1+\dfrac{2|\mathcal{A}|^2}{I_{\text{sat}}}}\right\}\mathcal{A}$$

$$r=\frac{g^{(0)}L_{\text{A}}}{T+\alpha}$$ Non saturated gain (normalized by losses)

$$\boxed{\frac{d\mathcal{A}}{dt}=\frac{\gamma_{\text{cav}}}{2}\left\{-1+\frac{\text{r}}{1+2|\mathcal{A}|^2/I_{\text{sat}}}\right\}\mathcal{A}}$$

Evolution equation of the intensity

$$\frac{d\mathcal{A}}{dt} = \frac{\gamma_{\text{cav}}}{2}\left\{-1+\frac{\mathrm{r}}{1+2\left|\mathcal{A}\right|^2/I_{\text{sat}}}\right\}\mathcal{A}$$

Amplitude and phase: $\mathcal{A} = A\mathrm{e}^{i\phi}$

no equation for the phase

$$\frac{dA}{dt} = \frac{\gamma_{\text{cav}}}{2}\left\{-1+\frac{\mathrm{r}}{1+2A^2/I_{\text{sat}}}\right\}A$$

Intensity

$I \propto \left|\mathcal{A}\right|^2 = A^2$

$$\frac{dI}{dt} = \gamma_{\text{cav}}\left\{-1+\frac{\mathrm{r}}{1+I/I_{\text{sat}}}\right\}I$$

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Steady state (stationnary) intensity

$$\frac{dI}{dt} = \gamma_{\text{cav}} \left\{ -1 + \frac{\text{r}}{1 + I / I_{\text{sat}}} \right\} I = 0 \quad \Longrightarrow \quad \boxed{\begin{aligned} I' &= 0 \\ I'' &= (r-1) I_{\text{sat}} \end{aligned}}$$

Non zero solution only if $r = \dfrac{g_0 L_{\text{A}}}{T + \alpha} > 1$

Non saturated gain > losses

If $r > 1$: There exists two solutions!

Which of the two is selected by the laser?

Stability of the stationnary solutions

Linearization of $\frac{dI}{dt} = \gamma_{\text{cav}} \left\{ -1 + \frac{\text{r}}{1 + I / I_{\text{sat}}} \right\} I$ in the vicinity of solutions

- Stability of $\boxed{I'' = (r-1) I_{\text{sat}}}$

Noting $I = I'' + i$

one expands at 1rst order in i / I''

  - Order 0 is automatically satisfied (stationnary solution)
  - Order 1: $\frac{di}{dt} \sim -\gamma_{\text{cav}} \left(\frac{r-1}{r} \right) i$

Case $r > 1$ ($I'' \neq 0$), stable solution: $I = I'' + \delta I \exp\left\{ -\gamma_{\text{cav}} \left(\frac{r-1}{r} \right) t \right\}$

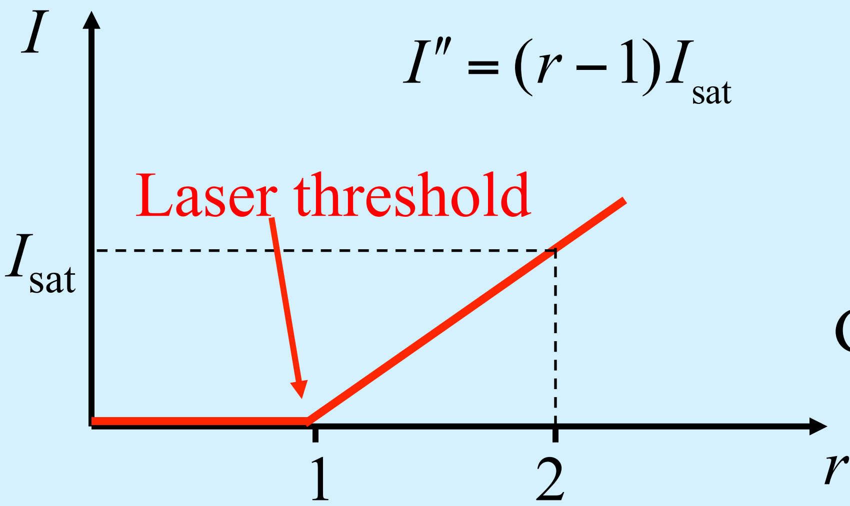
- Stability of $I' = 0$
  - stable if $r < 1$
  - unstable $r > 1$

Above threshold, non zero solution!
Starts on a spontaneous photon

Stable stationnary solutions

- $r < 1$: $I' = 0$
- $r > 1$: $I'' = (r-1) I_{\text{sat}}$

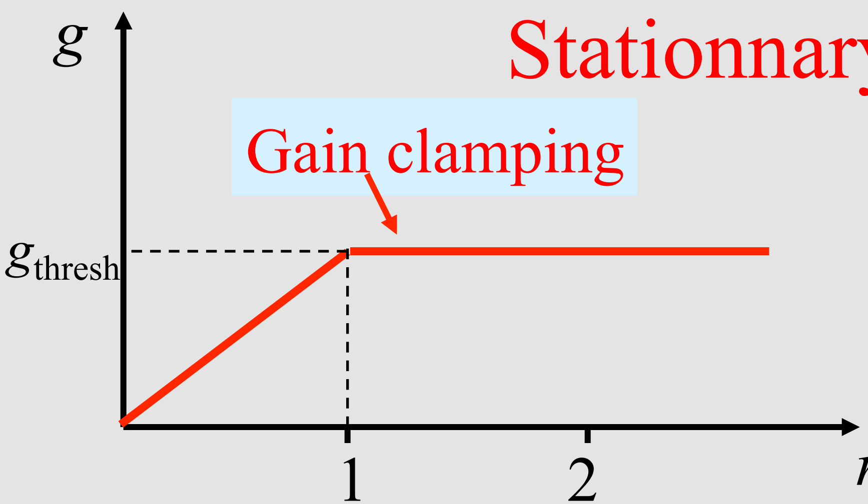
Stationnary intensity and gain



$$r = \frac{g^{(0)} L_A}{T + \alpha}$$ « Excitation » of amplifier

Gain at threshold (non saturated) $$g_{thresh} = \frac{T + \alpha}{L_A}$$

Stationnary gain



$$g = \frac{g^{(0)}}{1 + I / I_{sat}}$$

- $r < 1 \ : \ I = 0$
- $r > 1 \ : \ I = (r-1)I_{sat}$

Saturation prevents gain from exceeding losses, in order to fulfill the steady state condition *gain = losses*

Laser threshold and phase transition

$$\boxed{\left(-1+\frac{r}{1+2\left|\mathcal{A}\right|^2/I_{\text{sat}}}\right)\mathcal{A}=0}$$

has non zero solutions only if $r>1$

$\boxed{\text{A complex field (order parameter) appears at } r=0}$

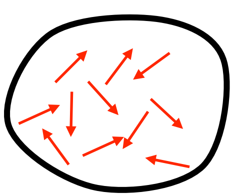
Analogy with a phase transition

Ferromagnetic medium at Curie point: spontaneous magnetization appears at Curie temperature T_C

$$\boxed{c\left(T-T_C\right)\mathbf{M}+gT\left|\mathbf{M}\right|^2\mathbf{M}=0}$$

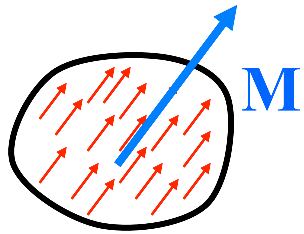
has non zero solutions only if $T<T_C$

$T>T_C$



$\mathbf{M}=0$

$T<T_C$

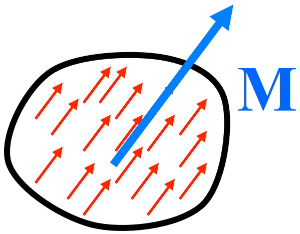


$\mathbf{M}=0$ is unstable

$$\left|\mathbf{M}\right|^2=\frac{c\left(T_C-T\right)}{gT}$$

Spontaneous symetry breaking

$T < T_C$: appearing of a magnetization **M** (order parameter)



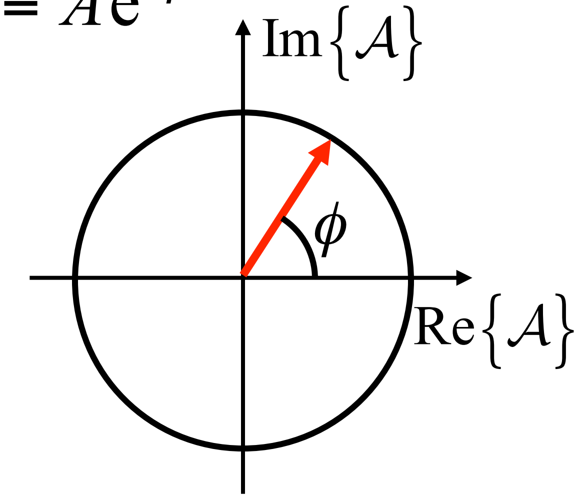
$$|\mathbf{M}|^2 = \frac{c\left(T - T_C\right)}{gT}$$

does not determine the direction of **M** ! ??

Spontaneous symetry breaking : a direction is chosen

All elementary dipoles take the same orientation

$r > 1$: appearance of a complex laser field $\mathcal{A} = A\,\mathrm{e}^{i\phi}$



$$|\mathcal{A}|^2 = (r-1)\frac{I_{\mathrm{sat}}}{2}$$

does not determine the phase of the field

Spontaneous symetry breaking: a phase is chosen

All emitters locked in the same phase

Violation of the Curie principle?

Curie principle

- Effects have the same symmetry as causes
- Solutions of a problem have the same symetry as the physical situation (equations, boundary conditions)

Magnetization : situation *a priori* invariant by rotation in space

The solution picks up a direction !

Laser field: problem invariant in the Fresnel plane (phasor plane)

The solution picks up a phase

A catalogue of possible reactions to the conflict

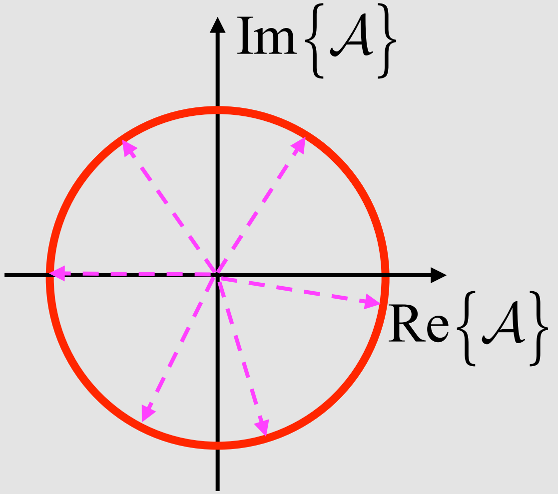
1. Theoretical : Spontaneous symmetry breaking does exist (and it is a useful and fruitful concept!)
2. Pragmatic: an initial residual field breaks the symetry
3. Formal: search a solution as a random variable respecting the symmetry

The complex amplitude of the laser mode as an classical random variable

We look for a solution of $|\mathcal{A}|^2 = (r-1)\, I_{sat} / 2$ as a classical complex random variable.

$\mathcal{A} = A_1 \mathrm{e}^{i\phi}$ = random variable such as

- $A_1 = \sqrt{(r-1)\dfrac{I_{sat}}{2}}$
- ϕ uniformly distributed over $[0, 2\pi]$



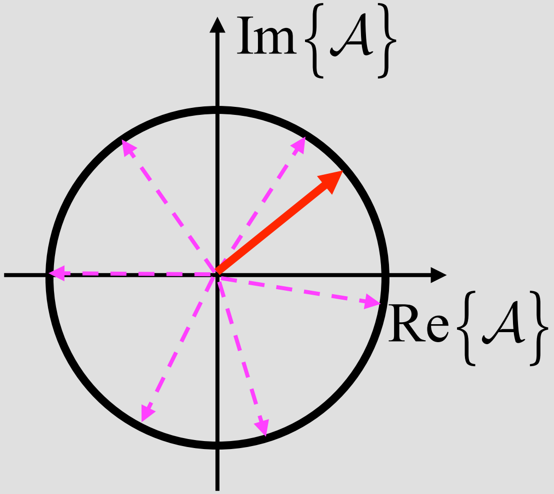
- is a solution
- does not break the symetry

What happens for a specific situation? (a particular laser has been turned on).

Complex amplitude of the laser mode in a particular experiment: a realization (a sample) of the random variable

$\mathcal{A} = A_1 \mathrm{e}^{i\phi}$ = random variable such as

- $A_1 = \sqrt{(r-1)\dfrac{I_{\mathrm{sat}}}{2}}$
- ϕ uniform over $[0, 2\pi]$



A particular state of working: a particular realization of the random variable (a sample drawn from a statistical ensemble): one particular value of the phase is picked

Powerfull method: ensemble averages allow one to obtain results on a specific situation! (cf. lectures 6 and 7)

Description in the spirit of full quantum optics formalism: quantum state without a specific phase; measurement determines the phase.

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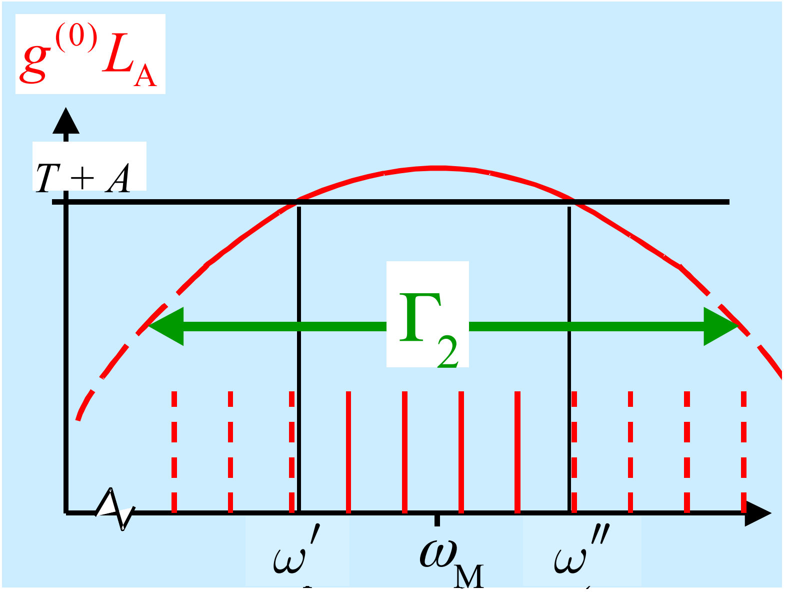
Saturation in a multimode laser?

Monomode laser gain

$$g(\omega_p) = \frac{g^{(0)}(\omega_p)}{1 + \dfrac{I_p}{I_{\text{sat}}(\omega_p)}}$$



Multimode laser

How to write the saturation term relative to mode p when several modes are active?

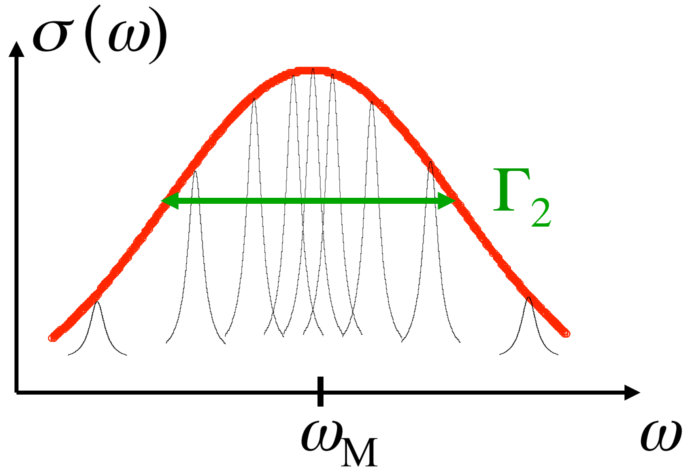
$$\frac{I_p}{I_{\text{sat}}} \quad \text{or} \quad \frac{\sum_q I_q}{I_{\text{sat}}} \;?$$

Depends on the nature of the broadening of the laser line, of width Γ_2:

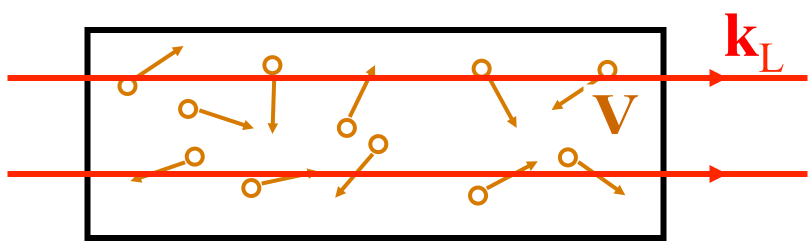
- Homogeneous broadening (same for all atoms)
- Inhomogeneous broadening (atoms with different properties)

Inhomogeneous broadening

The overall line width results from the addition of individual lines centered at different frequencies, associated with different microscopic amplifying systems



Example: Doppler broadening in a gas laser (*eg* He-Ne)



Doppler shift for an atom at velocity **V** interacting with a light wave with wave vector $\mathbf{k}_L$

$$\delta\omega = \mathbf{k}.\mathbf{V} = \omega_L \frac{V_{\mathbf{k}_L}}{c}$$

Doppler broadening:

$$\boxed{\Delta\omega_D = \omega_L \frac{\Delta V_T}{c} \gg \text{ individual broadening}}$$

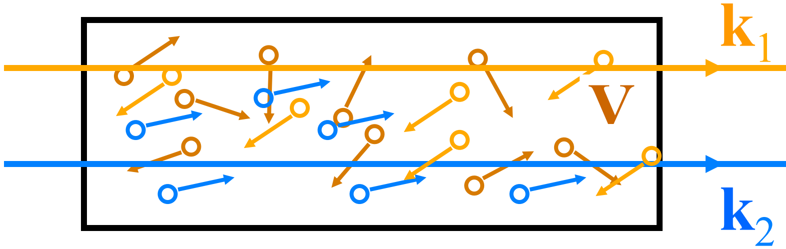
$$\Rightarrow \boxed{\Gamma_2 \simeq \Delta\omega_D}$$

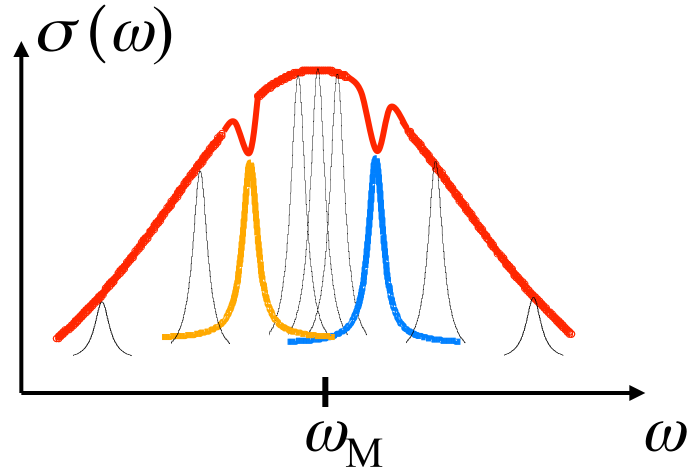
$\approx$ GHz ($\Delta V_T \approx 10^3$ m / s)

Saturation in the inhomogeneous broadening case

Waves with different frequencies ω_1 , ω_2

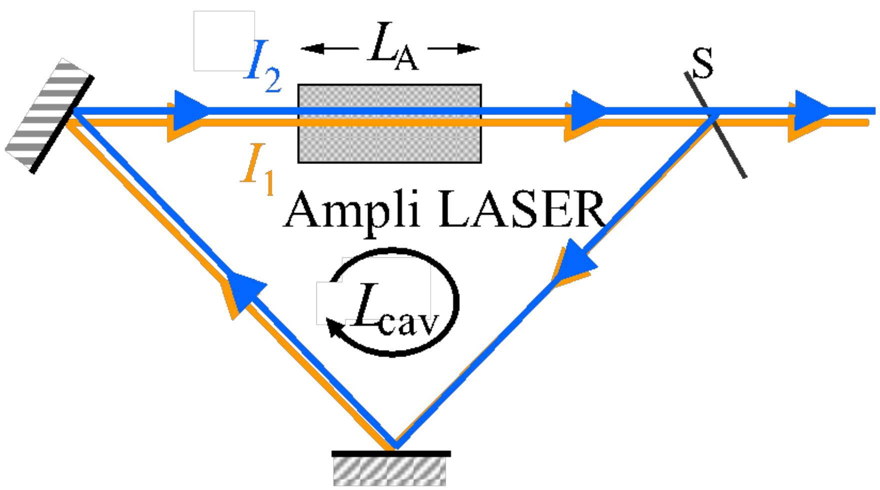
Interact with different atoms

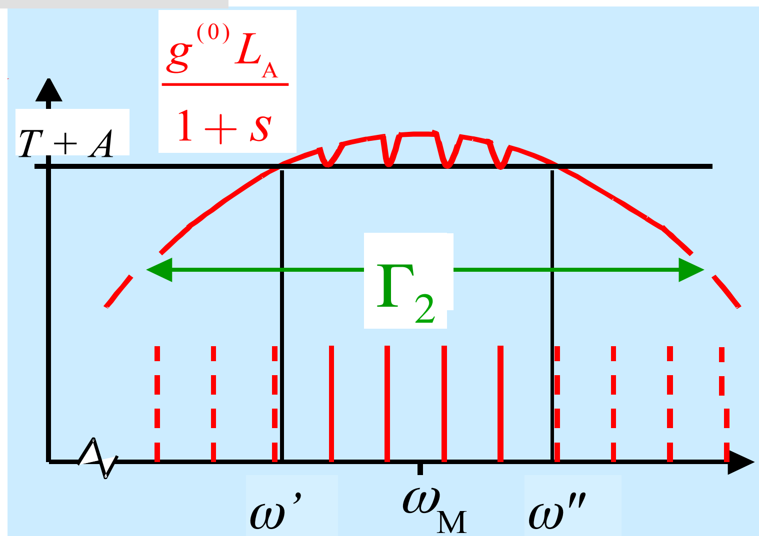




No cross saturation: each mode behaves as an independant laser.

Multimode behaviour





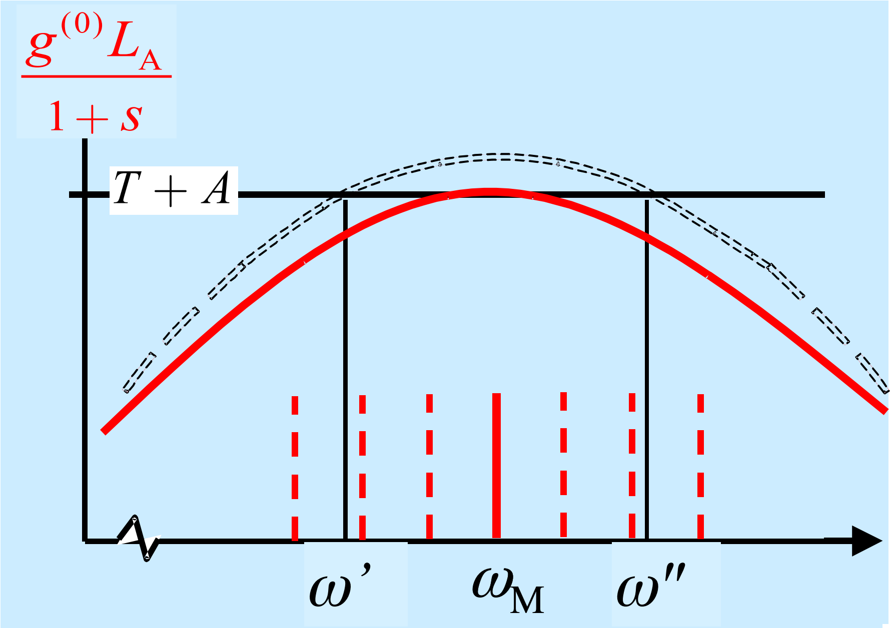
Saturation in the homogeneous broadening case

All atoms behave the same way.

The total line is identical to each (broad) individual line.

Each atom is saturated by the total intensity $\sum_q I_q$

All modes saturated similarly



Because of saturation, one mode only can be active

Mode competition: 1 mode only survives; monomode behaviour

Examples : Nd:YAG; high pressure CO_2; semi conductor lasers

Mixed situation

Most often, intermediate situation: some degree of cross saturation

$$\boxed{g(\omega_p) = \frac{g^{(0)}(\omega_p)}{1 + \sum_q \frac{\beta_{pq} I_q}{I_{\text{sat}}}}}$$

- Pure homogeneous case: $\beta_{pq} = \delta_{pq}$
- Pure inhomogeneous case: $\beta_{pq} = 1$

Evolution of each mode

$$\frac{dI_p}{dt} = \gamma_{\text{cav}} \left(-1 + \frac{r}{1 + \sum_q \frac{\beta_{pq} I_q}{I_{\text{sat}}}} \right) I_p$$

⇨ Moderated saturation regime

$$\boxed{\frac{dI_p}{dt} \simeq \gamma_{\text{cav}} (r-1) I_p - \gamma_{\text{cav}} r \frac{\beta_{pp}}{I_{\text{sat}}} I_p^{\,2} - \gamma_{\text{cav}} r \sum_{q \neq p} \frac{\beta_{pq}}{I_{\text{sat}}} I_q I_p}$$

gain − losses

Self saturation

Cross saturation

Example of two modes partially coupled

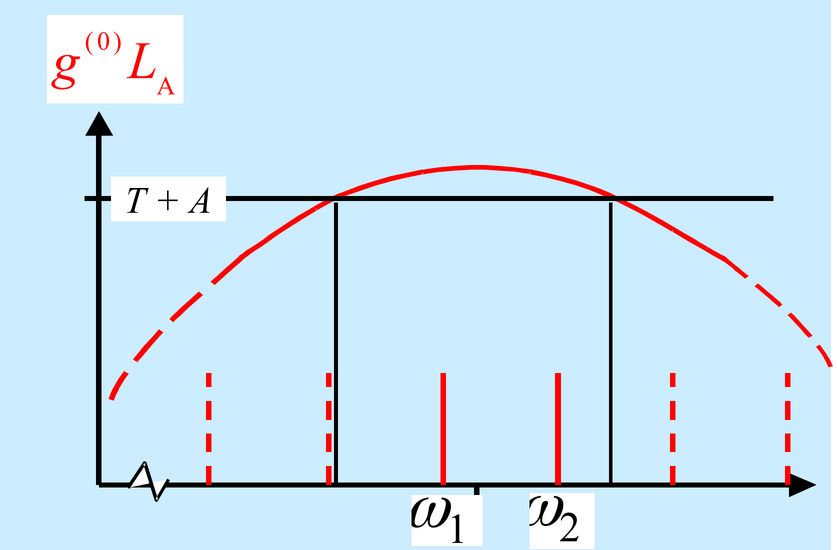
$$\frac{dI_1}{dt} = \alpha_1 I_1 - \beta_1 I_1^{\,2} - \theta_{12} I_1 I_2$$

$$\frac{dI_2}{dt} = \alpha_2 I_2 - \beta_2 I_2^{\,2} - \theta_{21} I_1 I_2$$

gain − losses $\gamma_{\text{cav}}(r-1)$

Self saturation $-\gamma_{\text{cav}} r \beta_{22} / I_{\text{sat}}$

Cross saturation $\gamma_{\text{cav}} r \beta_{21} / I_{\text{sat}}$



Stationnary solutions

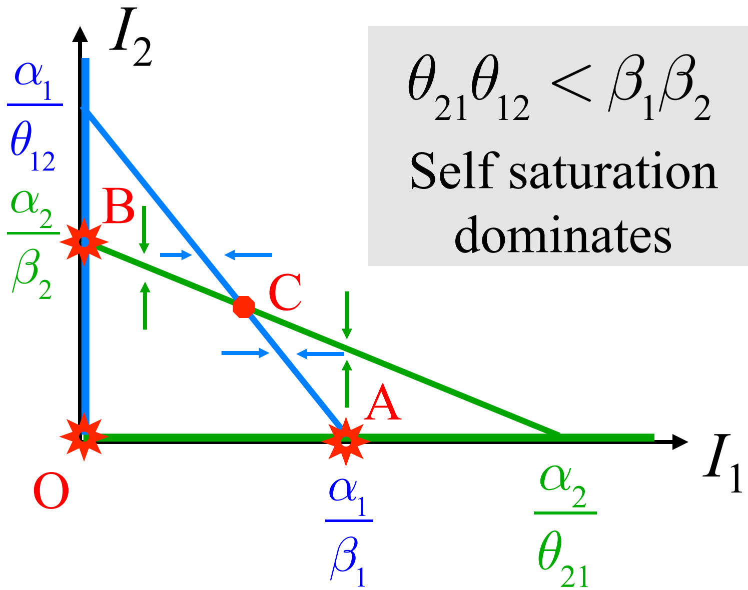
$$\frac{dI_1}{dt} = 0 \Rightarrow \begin{cases} I_1 = 0 \\ \alpha_1 - \beta_1 I_1 - \theta_{12} I_2 = 0 \end{cases} \quad (1)$$

$$\frac{dI_2}{dt} = 0 \Rightarrow \begin{cases} I_2 = 0 \\ \alpha_2 - \beta_2 I_2 - \theta_{21} I_1 = 0 \end{cases} \quad (2)$$

We look for solutions common to (1) and (2)

Graphic method
$\Rightarrow$ several cases to be distinguished

Two active modes



$\theta_{21}\theta_{12} < \beta_1\beta_2$

Self saturation dominates

$$(1) \begin{cases} I_1 = 0 \\ \alpha_1 - \beta_1 I_1 - \theta_{12} I_2 = 0 \end{cases}$$

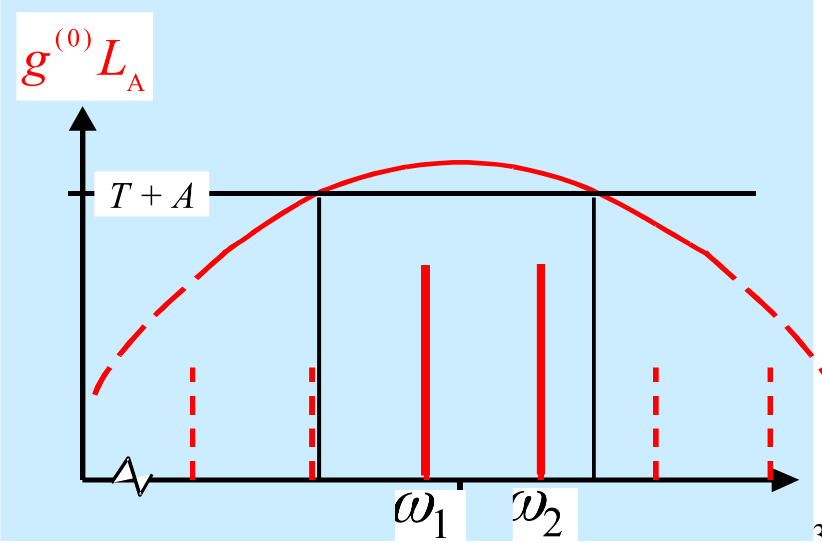
$$(2) \begin{cases} I_2 = 0 \\ \alpha_2 - \beta_2 I_2 - \theta_{21} I_1 = 0 \end{cases}$$

Inhomogeneous broadening case

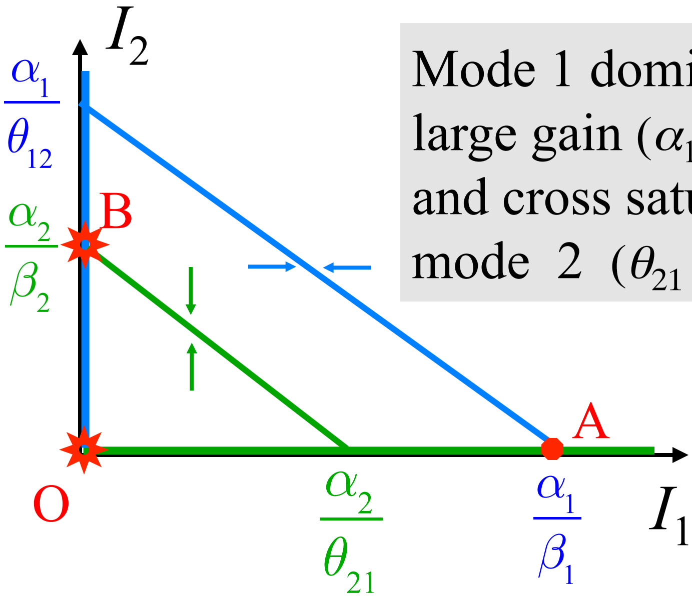
Solutions belonging to (1) and (2): blue-green intersections

Stability study: symbols indicate the result

- O, A, B are unstables
- C is stable: $I_1 \neq 0$; $I_2 \neq 0$
  two mode behaviour



Mode competition (simple)



Mode 1 dominates: large gain (α_1 « big ») and cross saturates mode 2 (θ_{21} « big »)

$$(1)\ \begin{cases} I_1 = 0 \\ \alpha_1 - \beta_1 I_1 - \theta_{12} I_2 = 0 \end{cases}$$

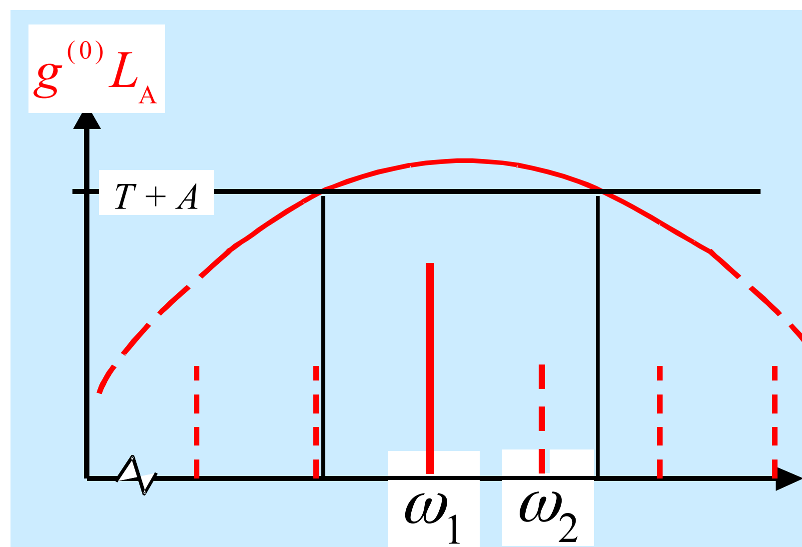
$$(2)\ \begin{cases} I_2 = 0 \\ \alpha_2 - \beta_2 I_2 - \theta_{21} I_1 = 0 \end{cases}$$

Homogeneous case with dominating mode

Solutions belonging to (1) and (2) : blue-green intersections O, A, B

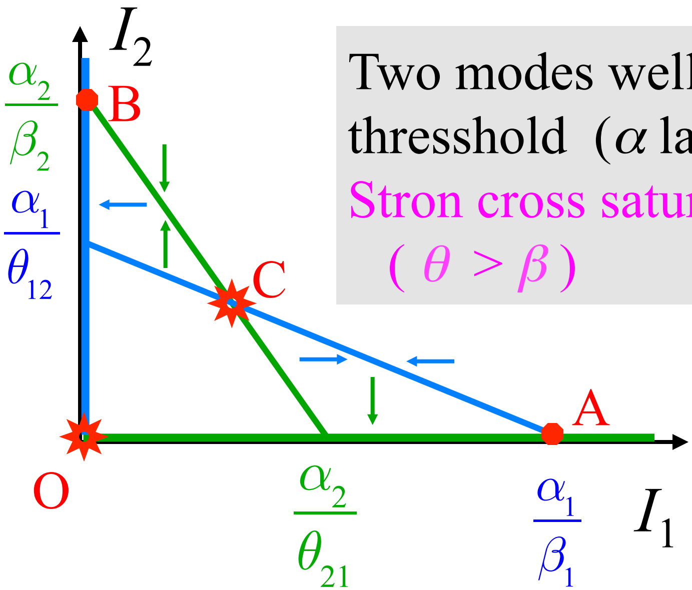
Stability studies (see notes)

- O, B are unstable
- A is stable: only mode ω_1 is active



The more favored mode ‘kills’ the less favored mode: simple mode competition

Mode competition: bistable behaviour



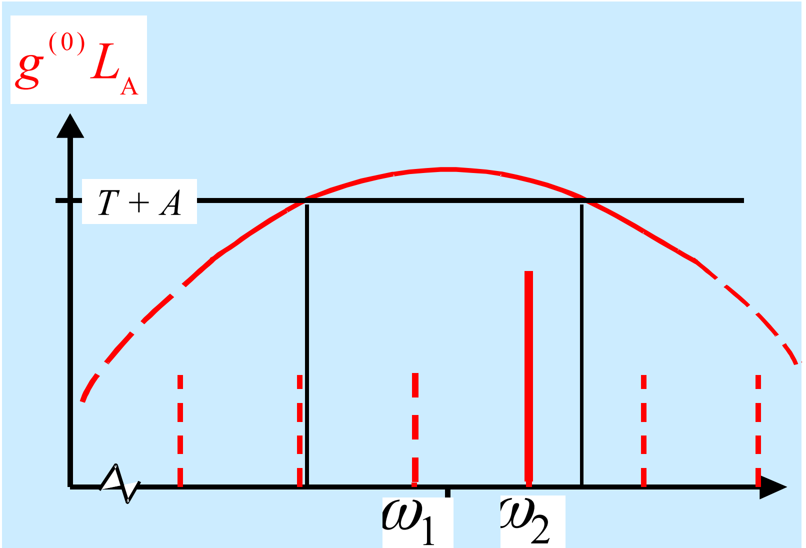
Two modes well above thresshold (α large)
Stron cross saturation
($\theta > \beta$)

$$(1)\ \begin{cases} I_1 = 0 \\ \alpha_1 - \beta_1 I_1 - \theta_{12} I_2 = 0 \end{cases}$$

$$(2)\ \begin{cases} I_2 = 0 \\ \alpha_2 - \beta_2 I_2 - \theta_{21} I_1 = 0 \end{cases}$$

Homogeneous case with dominant mode

Stability study (cf notes) :
2 stable points! Two distinct solutions Are possible.

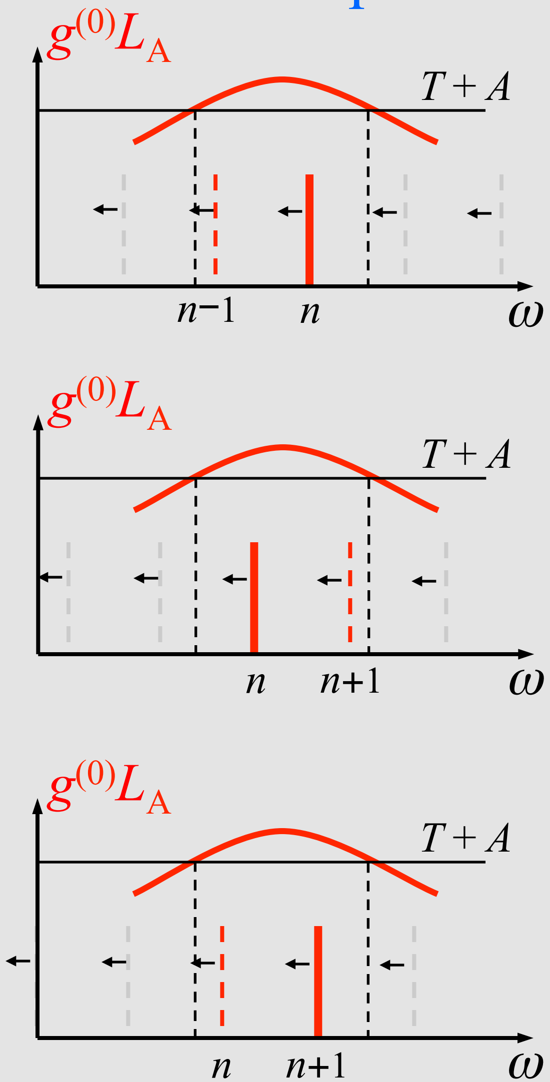


What is the choice of the system?
Depends on previous situation:
system with a memory.

Applications: memory to store information

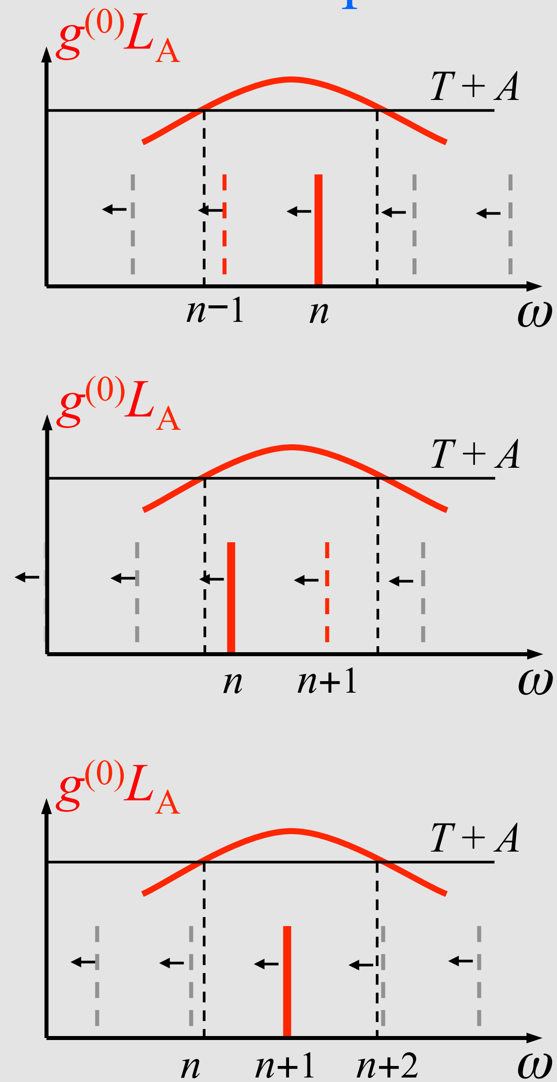
Mode competition: simple *vs.* bistable

Compétition simple



The most favoured mode always wins

Compétition bistable



The already estabished mode is still dominant even when it becomes less favoured (provided gain larger than losses: hystérésis

Robust: applications

Scenario with mode comb drifting to the left (cavity expansion)

Laser in stationnary state
Continuous Wave (CW) laser

A. Mode of the "cold" cavity
   - Closed lossless cavity
   - Open lossless cavity
   - Top hat mode model
   - Lossy cavity
   - Number of photons

B. Evolution of a mode amplitude
   - Gain; saturation
   - Evolution equation
     - •Amplitude
     - •Intensity

C. Single mode laser: stationary state
   - Stationary solutions
   - Intensity and gain
   - Threshold and phase transition
   - Spontaneous symmetry breaking

D. Multimode laser: mode competition
   - Homogeneous and inhomogeneous broadening
   - Multimode emission; simple or bistable mode competition

E. Conclusion
   - Generality of the behaviors

Conclusion

In that lecture

- Mode expansion: remarkable simplification
- Mode evolution with saturation term
- Stationary state (single mode): threshold, gain saturation, spontaneous symmetry breaking
- Two modes: mode competition, bistability

Non trivial behavior due to non-linear terms

- Saturation; threshold; clamping; spontaneous symmetry breaking
- Cross terms: competition; bistability, hysteresis
- Rich dynamics (transition to chaos, beyond that lecture)

General and generic phenomena

- Analogous behaviors (Volterra equations) in mechanics, biology, chemistry, economy…
- Laser physics: a remarkable play ground to study non-linear phenomena (phase transitions, transition to chaos, rogue waves…).