Solving Systems of Phaseless Equations with Optimal Complexity

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We consider the problem of solving systems of phaseless equations $|\langle a_i, x\rangle|^2 = y_i$, $i = 1, \ldots, m$ and x in $\mathbb{R}^n$ is unknown. One application of great importance is the phase retrieval problem, which provides promising and indispensable tools in a wide spectrum of techniques including X-ray crystallography, d-iffraction imaging, microscopy, and quantum mechanics. We will present two results when when the number of equations is proportional to the number of unknowns. (1) We will present a Riemannian gradient descent algorithm and a truncated variant. The algorithms are developed by exploiting the inherent low rank structure of the problem based on the embedded manifold of rank-1 positive semidefinite matrices. Theoretical recovery guarantee has been established for the truncated variant, showing that the algorithm is able to converge to x or $-x$ linearly when $m = O(n)$. (2) We will present the global geometry of a new non-convex objective function for solving the phaseless equations. The new objective function is constructed via a least squares fitting to the phaseless equations with an activation function. We prove that this new objective function with $m = O(n)$ has a nice geometry — there is no spurious local minima. Therefore, any algorithm finding a local minimum will give a solution of the phaseless equations.